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DETERMINATION OF THE SPEED OF SOUND ALONG THE HUGONIOT IN A SHOCKED MATERIAL

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14. ABSTRACT

Two methods were investigated for approximating the speed of sound (using the Hugoniot or a compression ratio factor). Both methods were shown to have a high degree of accuracy at low pressures and correctly predict higher speeds of sound for the higher energy shocked states. The approximations of higher shock pressures diverge progressively greater at higher shock states.

A relationship between the partial derivatives of the change in pressure to the change in volume along the Hugoniot to the isentrope was developed. The linear u_s-u_p and Mie-Grüneisen equations of state were applied to construct a comparison of the computed speeds of sound using copper.

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INTRODUCTION

A common approximation for the speed of sound has been to use the change in pressure with a change in volume along the Hugoniot as equal to the change in pressure with volume along the isentrope. This approximation has been used successfully but is not strictly correct. In order to evaluate the accuracy, the following calculations comparing the partial derivatives of pressure to volume along the isentrope and Hugoniot were conducted.

COMPUTATIONS

The speed of sound can be calculated using the partial derivative of pressure with specific volume along the isentrope by rearranging the definition of the speed of sound in its most common form: the partial derivative of pressure with density. From the definition of the speed of sound:

$$c = \sqrt{\left.\frac{\partial P}{\partial \rho}\right|_s} \tag{1}$$

The density is related to the specific volume by:

$$\rho = \frac{1}{v} \tag{2}$$

$$\partial \rho = -\frac{1}{v^2}\partial v \tag{3}$$

Then:

$$c = \sqrt{-v^2 \left.\frac{\partial P}{\partial v}\right|_s} \tag{4}$$

Pressure is a state property and therefore can be determined from a function of two other state properties. In this development, specific volume and energy are being used.

$$P = f(v, e) \tag{5}$$

Then by the chain rule:

$$dP = \left.\frac{\partial P}{\partial v}\right|_e dv + \left.\frac{\partial P}{\partial e}\right|_v de \tag{6}$$

Dividing by dv:

$$\frac{dP}{dv} = \left.\frac{\partial P}{\partial v}\right|_e \frac{dv}{dv} + \left.\frac{\partial P}{\partial e}\right|_v \frac{de}{dv} \tag{7}$$

Which is the general solution and must be true along the Hugoniot:

$$\left.\frac{\partial P}{\partial v}\right|_H = \left.\frac{\partial P}{\partial v}\right|_e + \left.\frac{\partial P}{\partial e}\right|_v \left.\frac{\partial e}{\partial v}\right|_H \tag{8}$$

Or

$$\left.\frac{\partial P}{\partial v}\right|_e = \left.\frac{\partial P}{\partial v}\right|_H - \left.\frac{\partial P}{\partial e}\right|_v \left.\frac{\partial e}{\partial v}\right|_H \tag{9}$$

Developed by similar methods

$$\left.\frac{\partial P}{\partial v}\right|_s = \left.\frac{\partial P}{\partial v}\right|_e + \left.\frac{\partial P}{\partial e}\right|_v \left.\frac{\partial e}{\partial v}\right|_s \tag{10}$$

$$\left.\frac{\partial P}{\partial v}\right|_s = \left.\frac{\partial P}{\partial v}\right|_H + \left.\frac{\partial P}{\partial e}\right|_v \left(\left.\frac{\partial e}{\partial v}\right|_s - \left.\frac{\partial e}{\partial v}\right|_H\right) \tag{11}$$

Equation 11 relates $\left.\frac{\partial P}{\partial v}\right|_s$ to $\left.\frac{\partial P}{\partial v}\right|_H$, specifically the deviation between the two derivatives given by $\left.\frac{\partial P}{\partial e}\right|_v \left(\left.\frac{\partial e}{\partial v}\right|_s - \left.\frac{\partial e}{\partial v}\right|_H\right)$. Applying definition of entropy simplifies the computations.

From the fundamental thermodynamics relation:

$$de = Tds - Pdv \tag{12}$$

Along an isentrope, ds=0

$$-P = \left.\frac{\partial e}{\partial v}\right|_s \tag{13}$$

Further simplifications can be made by using an equation for the energy along the Hugoniot:

$$e = \frac{1}{2} P(v_0 - v) \tag{14}$$

$$\left.\frac{\partial e}{\partial v}\right|_H = \frac{1}{2}\left[\left.\frac{\partial P}{\partial v}\right|_H (v_0 - v) - P\right] \tag{15}$$

$$\left.\frac{\partial P}{\partial v}\right|_s = \left.\frac{\partial P}{\partial v}\right|_H + \left.\frac{\partial P}{\partial e}\right|_v \frac{1}{2}\left(-P - \left[\left.\frac{\partial P}{\partial v}\right|_H (v_0 - v)\right]\right) \tag{16}$$

The development to this point used the laws of thermodynamics and the conservation equations. As such, equation 12 will hold regardless of the equations of state used. Utilization of equations of state can further simplify this equation.

From the Mie-Grüneisen equation of state:

$$\frac{\gamma}{v} = \left.\frac{\partial P}{\partial e}\right|_v = a\ constant \tag{17}$$

Then

$$\left.\frac{\partial P}{\partial v}\right|_s = -\frac{\gamma}{v}\frac{P}{2} + \left.\frac{\partial P}{\partial v}\right|_H \left[1 - \frac{\gamma}{v}\frac{1}{2}(v_0 - v)\right] \tag{18}$$

Using the linear equation of state, u_s-u_p:

$$u_s = c_0 + u_p \tag{19}$$

Which when combined with the application of mass, momentum, and energy, the pressure of a shockwave is calculated as:

$$P = \frac{{c_0}^2(v_0 - v)}{[v_0 - s(v_0 - v)]^2} \tag{20}$$

$$\left.\frac{\partial P}{\partial v}\right|_H = -{c_0}^2\frac{v_0 + s(v_0 - v)}{[v_0 - s(v_0 - v)]^3} \tag{21}$$

Using equations 14 and 16, a comparison can be made between the variables. Copper was selected as a basis for comparison, and the following variables were used as shown in table 1 (ref. 1):

Table 1
Comparison between variables

Density	8930	kg/m^3
Ambient speed of sound	3940	m/s
Linear u_s-u_p coefficient (s)	1.489	unitless
Mie-Grüneisen gamma, approximately = 2s-1 (ref. 2)	1.978	

For comparison, the pressure (fig. 1) and energy (fig. 2) were plotted against volume after the shock compaction.

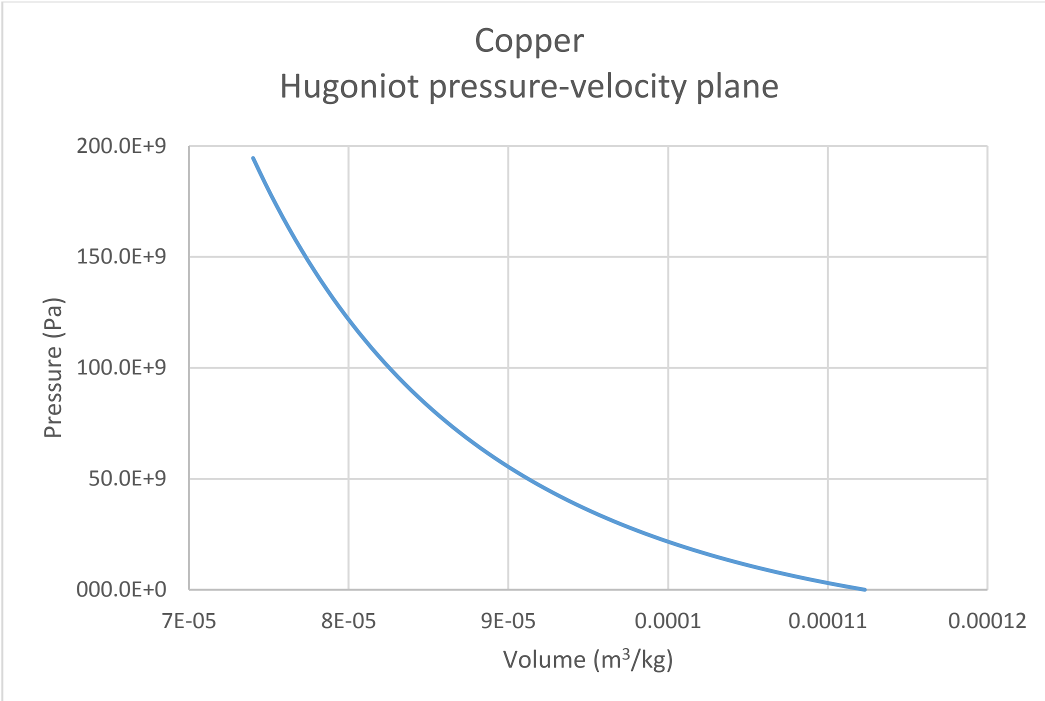

Figure 1
Copper Hugoniot pressure-specific volume plane

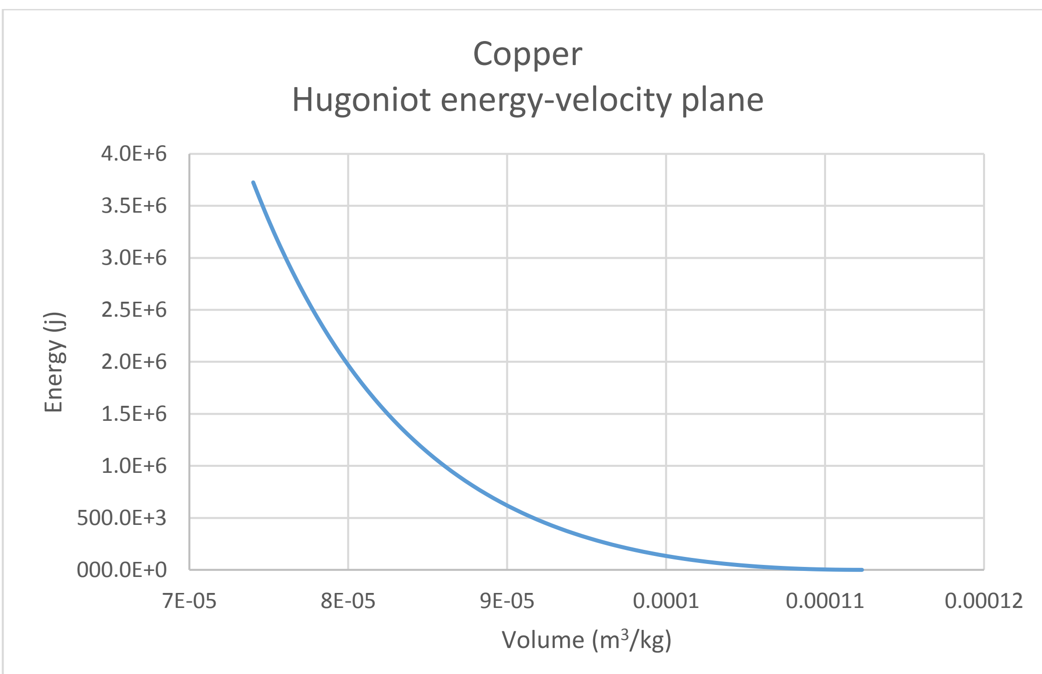

Figure 2
Copper Hugoniot energy-specific volume plane

Figure 3 shows a comparison between the rate of change of the pressure with volume along the isentrope and the Hugoniot. These derivatives are identical at the initial volume and diverge a greater amount for progressively higher compressions. Figure 4 uses the data from figure 3 to compute the speed of sound in the shocked materials.

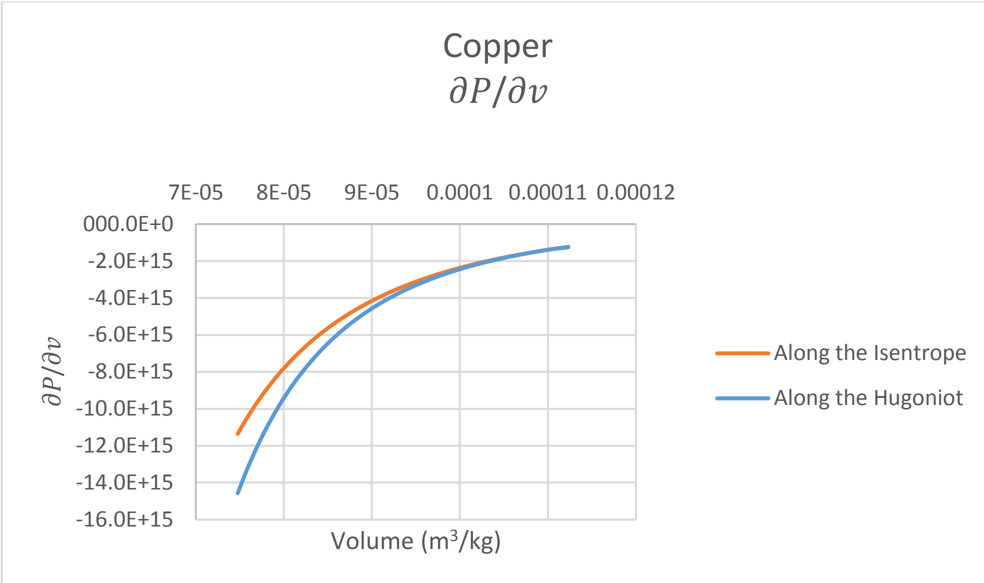

Figure 3
Comparison between rate of change of pressure with volume along isentrope and Hugoniot in copper

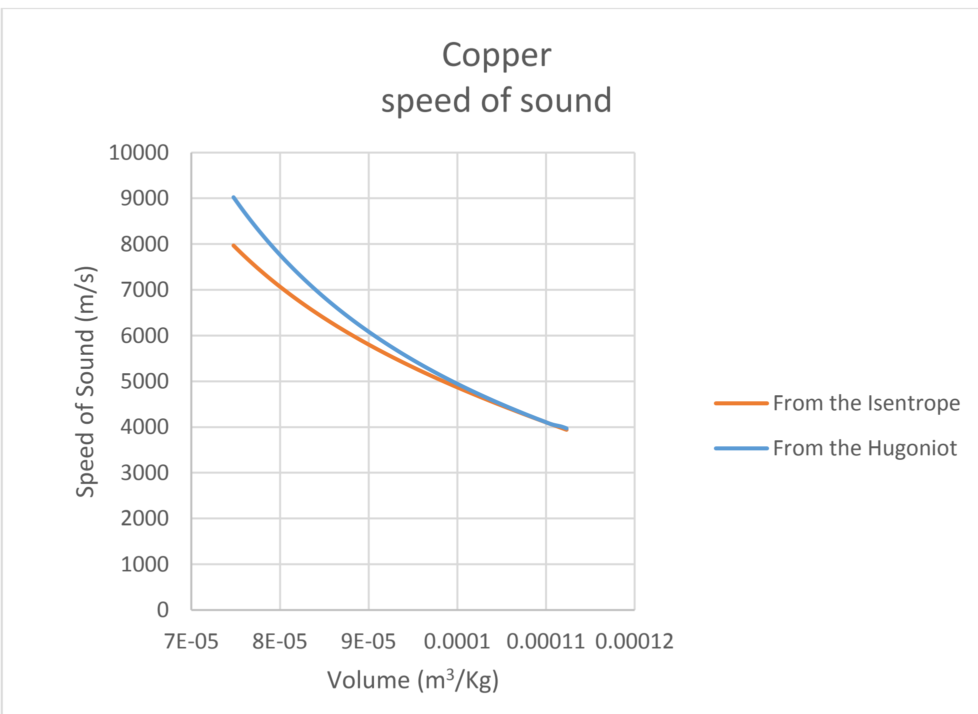

Figure 4
Speed of sound in shocked copper

It has been proposed to use the compression ratio to increase the speed of sound for the compressed material. Figure 5 compares this method to using the Hugoniot for computing the speed of sound. This method shows a greater deviation from the computed speed of sound when compared to the utilization of the Hugoniot.

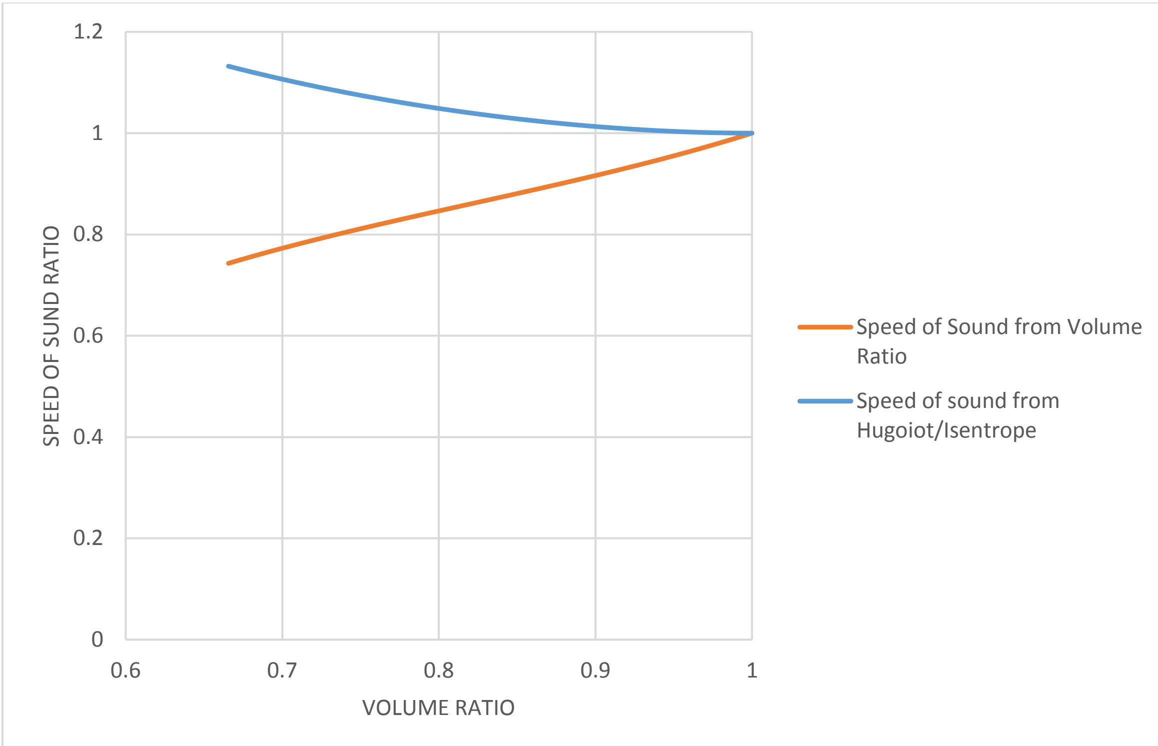

Figure 5
Copper ratio of speeds of sound

CONCLUSIONS

The methods investigated for approximating the speed of sound (using the Hugoniot or a compression ratio factor) have a high degree of accuracy at low pressures and correctly converge to the ambient speed of sound. All methods correctly predict higher speeds of sound for the higher energy shocked states. The approximations diverge progressively greater at higher shock states.

A number of equations were developed that relate the partial derivatives of the change in pressure to the volume along the Hugoniot to the isentrope. These equations are correct only for states on the isentrope (those obtained as a function of a single shock). The first uses only thermodynamics and the conservation equations and, as such, would be universally applicable. The second assumes an equation of state that allows for the computations of the values for the speed of sound.

REFERENCES

1. Cooper, P.W., "Explosive Engineering," Wiley-VCH, 1996.
2. Meyers, M. A., "Dynamic Behavior of Material," John Wiley & Sons, Inc., 1994.

LIST OF SYMBOLS

Variables

c	Speed of sound
e	Specific internal energy
P	Pressure
s	Entropy
u_s	Shock velocity
u_p	Particle velocity
v	Specific volume
γ	Mie-Grüneisen coefficient
ρ	Density

Subscripts

H	Hugoniot
s	Isotopic
e	Constant energy
v	Isochoric
0	Initial state

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